**NU Industries Production Maximization**

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2021FA-MSDS\_460-DL\_SEC57

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December 5, 2021

**Abstract**

An optimization problem is formulated to *maximize NU industries* objective function of maximizing its profit with respect to its constraints related to various areas of the entire process using a linear programming model. Linear programming is a problem-solving method that has been utilized for decades in various industries. One of its more prominent uses have been in manufacturing and engineering departments, where output must be optimized within the allowable parameters analyzed using the sensitivity analysis. Linear Programming allows decision makers to gain key insight on the impact of slight changes to variables when solving complex issues.

*Keywords: Optimization, Linear programming, sensitivity analysis, Production Scheduling*.

**Introduction**

Optimization is a discipline that can differentiate manufacturers in respect to their success. Optimization in manufacturing plants can allow one to maximize their operational capacity, while minimizing waste and variable costs to satisfy the company’s objective. NU Industries operates two manufacturing plants that specialize in making three different products: Widgets, Gadgets, and Flugels. Presently, the company has the capability to produce all three products at either plant which then get shipped to a distribution center as “finished goods” to be shipped to the end customers.

Each product requires a unique mix of raw materials and labor hours. Furthermore, even though both plants can make the same product, they each require a different raw material and labor hour mix for each product. This difference can be attributed to machinery difference, skilled labor, number of employees etc. Both plants also have access to an unbounded limit of overtime labor hours available based on business need.

Table

Description automatically generated

The Sales Team has provided *NU industries* with the above demand for the next five periods based on existing contracts, which needs to be met while maximizing profit and minimizing its costs. Aside from the existing demand, the marketing and forecasting team anticipates additional demand that can be created through effective advertising using the allocated budget of $70,000. The total advertising budget can be allocated to any period in part or in full depending on highest yield. Once demand is recognized for a given period, *NU Industries* needs to allocate production to its two plants. Each plant is allowed to produce products more than demand and carry the excess products to future periods with respect to its storage area. Plant A can store a maximum of 70 units, whereas Plant B can store up to 50 units. Furthermore, due to limited storage, only finished goods can be stored, raw material is only purchased based on quantity needed.

*NU Industries* currently sells its products at the following prices: Widgets for $2490, Gadgets for $1990, and Flugels for $2970. Our goal is to maximize NU industries profit with respect to its costs and advertising budgets. For this analysis, we will only be considering the variable costs and all fixed costs associated with Plant A and Plant B will be ignored. We will develop a linear programming model which will consider the following questions:

* What is the optimal production mix by period and by plant?
* How many units of each product should be produced in each period?
* Does one plant yield better profits than the other for any of the three products?
* Do transportation costs have a significant impact on a product's profitability?

Based on preliminary review as well as the information provided, we anticipate that the three products will be distributed unevenly between the two plants based on their raw material and labor requirements as well as their associated costs. To perform our analysis, we will use *excel* for modeling and its *solver* tool for analysis. Furthermore, we will also conduct a sensitivity analysis to test how slight changes in coefficients of our objective function impact the optimal solution.

**Literature Review**

There are several uses for linear programming that can be applied in various areas of the entire supply chain. Since the study done on *NU industries* was in the manufacturing world, we will be focusing on similar studies done to optimize various factors. Optimizing a supply chain does not just deal with the mix of the most profitable mix of units or the least amount of waste; it also relates to what type of units are being produced, or how much of a particular resource should be used to produce a particular unit. Furthermore, linear optimization can also be used to optimize the interdependency between two pieces to further increase ones’ capacity and increase the output.

Agriculture is one area where linear programming has been used for ages and will continue to do so in the future. In a study done by Mahak Bhatia, she discusses the various issues farmers face when deciding what crops to grow and how to allocate their land to maximize output while minimizing waste and risk. She discusses the “risk” factor by talking about the risk of disease on crops which can cause vegetation to die leading to increased losses. (Bhatia 2019) Furthermore, her study can be compared to the Pap Zoltans crop rotation linear programming model which was made in the late 2000s’ to maximize crop output by adopting crop rotation policy. (Zoltan 2008). The results of both studies for both revealed that the total income generated by linear programming model was higher than just simply using the crop rotation policy. This can be attributed to the fact that the linear programming model was able to take multiple things into account. (Bhatia 2019)

As more companies have started relying on data and started making data driven decisions, the use of robust optimization techniques for linear optimization as well as mixed linear optimization techniques have become common. Such methods and techniques account for the uncertainty of a data set through various metrics which can be seen to some degree when performing a sensitivity analysis. These techniques address the impact of uncertainty in the left hand side, right hand side, and in the objective function. These techniques started becoming more prevalent in refinery production planning and batch processing plants where multiple raw materials are used. The study revealed that by addressing the uncertainty of certain variables, refining plants were able to be more efficient and profitable in their output as well as the saving from the waste that got eliminated through better resource allocation. (Zukui Li, Ran Ding, and Christodoulos A. Floudas 2011). Another study done to address the multiple variable problem revealed that many generational techniques did in-fact fail to consider the total impact of slight changes in certain variables in a complex problem. The findings of the study showed that no single variable can be analyzed alone and must be looked at as one cohesive machine. (Michael, Wodzimierz 2002).

Since the internet boom, most consumer commerce activities have been moving virtual and organizations have had to become creative in how they attract new clients to their businesses. Due to this shift, sales promotions are crucial in any fast-moving consumer goods industry. In most cases, advertising and promotions prove to be costly making it crucial to understand the impact of such activities on sales and profit prior to making the investment. A study to understand the importance of successful promotions was done at grocery stores using linear integer programming to grasp the flow of sales with the introduction of promotions. The study resulted in a 3% increase with a potential increase of 5% by manipulating some of the constraints. (Maxime, Ngai-Hang, Kiran, Georgia, Anthony 2017).

All these studies are in whole or part like the study done on NU industries as they all aim at various topics covered in our project. Agriculture studies show the interconnectivity between various crops which is like our problem with two manufacturing plants having different requirements for each. Just like in an agricultural setting, finding the right mix is crucial to minimize costs, while maximizing output and sales. Furthermore, like most real-world applications today, multivariable models are a must as there is almost always a multitude of factors that directly impact the final output. Furthermore, promotions and advertisements can provide a boost to sales if done efficiently. Since it can be quite costly to invest in this area, it is important to be able to predict its impact with certainty before making the investment.

**Methodology**

Our Objective is to maximize our profit by determining the marketing, production, distribution, and inventory strategy while satisfying existing demand as well as the added demand from marketing activities. The approach we used in our model was to divide it up by period to see the activity by section as well as ensure all constraints are being met. Furthermore, we assumed there is no inventory at the start of period 1 as well as no inventory left over at the end of period 5. In our calculations. Since we have demand in each period, we will satisfy that demand using units produced in that period and/or by using units in our inventory from previous periods.

Given the information, our regular labor usage is limited to 2500 and 3800 hours for Plant A and B respectively, with an unbounded overtime available to be scheduled as needed. Furthermore, the amount of total finished goods inventories each plant can store in a given period is 70 units for Plant A and 50 units for Plant B (Widgets + Gadgets + Flugels). Since the storage space is limited, neither plant can store raw materials for a future period. Therefore, all raw material purchased must be used in the same period.

Our Model consists of 102 decision variables and 55 total constraints not including the non-negativity constraint. These constraints are distributed to satisfy the following areas: Production, Raw material 1 availability, Raw material 2 availability, Inventory storage per period, Advertising Budget, Plant A labor Hours, Plant B labor Hours, Plant A Overtime hours, and Plant B Overtime hours.

***Decision Variables:***

***Production Variables:***

WAi – Widgets produced at Plant A – Period I (where i can be 1,2,3,4,5)

WBi – Widgets produced at Plant B – Period I (where i can be 1,2,3,4,5)

GAi – Gadgets produced at Plant A – Period I (where i can be 1,2,3,4,5)

GBi – Gadgets produced at Plant B – Period I (where i can be 1,2,3,4,5)

FAi – Number of Flugels produced at Plant A – Period I (where i can be 1,2,3,4,5)

FBi – Number of Flugels produced at Plant B – Period I (where i can be 1,2,3,4,5)

***Inventory Variables:***

WIAj – Inventory of widgets at Plant A – j (where j can 1,2,3,4,5) based on storage periods

WIBj – Inventory of widgets at Plant B – j (where j can 1,2,3,4,5) based on storage periods

GIAj – Inventory of gadgets at Plant A – j (where j can 1,2,3,4,5) based on storage periods

GIBj – Inventory of gadgets at Plant B – j (where j can 1,2,3,4,5) based on storage periods

FIAj – Inventory of flugels at Plant A – j (where j can 1,2,3,4,5) based on storage periods

FIBj – Inventory of flugels at Plant B – j (where j can 1,2,3,4,5) based on storage periods

***Overtime Variables***

WOAk – Number of widgets produced at Plant A during overtime – k (where k can 1,2,3,4,5) based on storage periods

WOBk – Number of widgets produced at Plant B during overtime – k (where k can 1,2,3,4,5) based on storage periods

GOAk – Number of gadgets produced at Plant A during overtime – k (where k can 1,2,3,4,5) based on storage periods

GOBk – Number of gadgets produced at Plant B during overtime – k (where k can 1,2,3,4,5) based on storage periods

FOAk – Number of flugels produced at Plant A during overtime – k (where k can 1,2,3,4,5) based on storage periods

FOBk – Number of flugels produced at Plant B during overtime – k (where k can 1,2,3,4,5) based on storage periods

***Advertising Variables***

WAl – Number of additional units of widgets from advertising investment – l (where l can 1,2,3,4) based on storage periods

GAl – Number of additional units of gadgets from advertising investment – l (where l can 1,2,3,4) based on storage periods

FAl – Number of additional units of flugels from advertising investment – l (where l can 1,2,3,4) based on storage periods

Within our model, we calculated our optimal solution as a difference between revenue and cost. Below is the detail behind our optimal solution.

Optimal Solution = Revenue - Cost

***Revenue:***

2490(WA1 + WOA1 + WB1 + WOB1 + WA2 + WOA2 +WB2 + WOB2 + WA3 + WOA3 + WB3 + WOB3 + WA4 + WOA4 + WB4 + WOB4 + WA5 + WOA5 + WB5 + WOB5) +

1990(GA1 + GOA1 + GB1 + GOB1 + GA2 + GOA2 + GB2 + GOB2 + GA3 + GOA3 + GB3 + GOB3 + GA4 + GOA4 + GB4 + GOB4 + GA5 + GOA5 + GB5 + GOB5) +

2970(FA1 + FOA1 + FB1 + FOB1 + FA2 + FOA2 + FB2 + FOB2 + FA3 + FOA3 + FB3 + FOB3 + FA4 + FOA4 + FB4 + FOB4 + FA5 + FOA5 + FB5 + FOB5)

***Cost:***

11(9.5\*WA1 + 7.1\*GA1 + 11.1\*FA1 ) +

11(9.5\*WA2 + 7.1\*GA2 + 11.1\*FA2 ) +

11.55(9.5\*WA3 + 7.1\*GA3 + 11.1\*FA3 ) +

11.55(9.5\*WA4 + 7.1\*GA4 + 11.1\*FA4 ) +

11.55(9.5\*WA5 + 7.1\*GA5 + 11.1\*FA5 )

For our constraints, we split them out by period to ensure all requirements were being met. For production constraints, the sum of units produced during the given period as well as existing inventory from prior periods was set equal to or greater than the demand for each product. Raw Material (1 & 2) Labor and Inventory constraints were set to the sum total for the three products as less than or equal to the allowed amounts for any given period. Furthermore, for the advertising budget constraint, we set the sum of the total amount allocated in all periods less than or equal to the total of 70K. Lastly, for overtime labor hours, there was no min or max needed to we set that as greater than or equal to 0.

Upon identifying all the constraints, we then proceeded to add the coefficients for our variables which included a combination of both calculations as well as data provided directly from the sales team. The variables that needed their coefficients to be calculated were Production and Overtime variables, since each of these two areas needed input from several factors such as, revenue, labor, transportation, and delivery costs. Once all the coefficients and constraints were added to the model, we used excels’ solver analytics to get the optimal distribution of the mix for each period. The data was then double checked to ensure all constraints are being satisfied. Furthermore, to test the functionality of the model, we tried changing the coefficients to capture the change in each of our moving pieces to ensure all pieces of the model were functioning properly.

**Computational Experiment and Results**

After the model was run using solver, the optimal profit for *NU industries* comes out to $7,934,028.69. The sensitivity report that was produced from our model can be found in the spreadsheet. Looking at the constraints, the demand production for the periods have an allowable increase and decrease that indicate the range that is permitted where the current optimal solution should remain the current optimal solution and its corresponding shadow price should remain relevant as well. For example, the allowable increase of the widgets produced in Period 1 Plant B is 110.60. If the coefficient that represents that variable increases by $10, the current optimal solution remains the optimal solution. But if it increases by $120, the optimal solution may change and the model should be rerun. Similarly, the allowable decrease shows the range that the coefficient can decrease without changing the optimal solution, which in this case for the number of widgets produced in Period 1 Plant B is 1E+30.

One instance that can be seen is the Period 1 Constraint on widget production. There is an allowable increase of 9.037 and a shadow price of -298.96. This information describes that if there is an extra 1 to 9.037 units of widget demand, the optimal objective value will decrease $298.96 for every extra unit. The following formula can be used to calculate the new optimal objective value: current optimal objective value + (shadow price \* unit change). The demand production constraints for all 5 periods have either a negative or zero shadow price, indicating that it will either decrease or not do anything to the optimal price if there was an extra unit added to the constraint. On the other hand, if there was a decrease in widget production in Period 1 by 1 unit, the optimal value would increase by $298.96.

The Raw Material 1 availability for Period 1 constraint has an allowable increase of 2251.44 and the shadow price of 7.04. Therefore, an increase up to 2251.44 units is allowed without changing the optimal solution or shadow price. By adding one unit of Raw Material 1, the optimal value increases by 7.04. Similarly, the Raw Material 1 constraint for the other periods would also increase the optimal value for every extra unit of material. Increasing the units on the Raw Material 2 constraints, the optimal objective value results in an increase of $121.70, $94.43, $92.69, $120.35 for each extra unit in each period respectively. An increase of one unit in the inventory space will increase the value by approximately $393, $439, $281, $326 for Periods 1, 2, 3, and 4 respectively. The marketing budget will not increase the optimal value with an extra dollar because the shadow price is 0. The maximum labor hours for all of Plant B have an allowable increase of greater than 0. The shadow price is approximately 5 for those constraints. By increasing the maximum labor hours for Plant B, the optimal value will increase by each additional unit.

**Discussion and Conclusions**

The goal of this project was to maximize NU industries profit for five periods, which upon running the model netted a total of $7.9M with respect to all constraints (production, raw materials, marketing, transportation and labor). Furthermore, when analyzing the results by period, it appears that Plant A handled the majority of the production for Widgets and Gadgets, whereas, Plant B handled the production for Flugels. This is in line with the assumptions made at the beginning of the project based on the costs for each raw material, as well as the differences observed in the requirements for raw materials and labor for each product. Aside from the marketing and production assumptions, we also aimed to test the impact of transportation costs on the overall profitability; based on the results and output, there was no connection seen leading us to conclude that transportation costs is not a key variable in the overall model. Another interesting fact that can be observed is that the advertising budget remained unused, suggesting that investing in marketing for either of the three products would not yield increased profit.

The proposed output raises further questions and analysis to be done for future research, specifically regarding the constraints. Based on the sensitivity report, increasing or decreasing the right hand side of production and inventory constraints will have the biggest impact on the optimal value. As seen in the sensitivity report, an increase in just one unit of material in inventory stored from Period 1 to 2 allows the company to increase their overall profit by $393. Regular labor hours in Plant B, raw material 1, raw material 2, and inventory have shadow prices and an allowable increase greater than 0. So by increasing the units on their constraints, the optimal profit value will increase as well. Therefore, further research in that area could allow NU industries to further maximize their profit as well as explore investment opportunities to expand their plants. Investing in the existing plants can increase their ability to accommodate more regular labor hours, raw materials, as well as their storage area allowing for more units stored, or perhaps allow NU to store their raw materials as well to take advantage of slight price changes in costs.

From a technological standpoint, it would perhaps be beneficial to implement advanced analytical tools such as R or Python to better analyze the data. Since there were 102 variables and 55 constraints, using these advanced tools could simplify the analysis and allow for better visualizations. These two languages can also allow for A/B testing which can assist in analyzing the impact of individual variables on the overall output.

**Appendix**

***Demand Production Requirements:***

| **Product** | **Period 1** | **Period 2** | **Period 3** | **Period 4** | **Period 5** |
| --- | --- | --- | --- | --- | --- |
| Widgets | 70 | 125 | 185 | 190 | 200 |
| Gadgets | 200 | 300 | 295 | 245 | 240 |
| Flugels | 140 | 175 | 205 | 235 | 230 |

Plant A - The product inventory area can store a combined maximum of 70 units

Plant B - The product inventory area can store a combined maximum of 50 units

**Period 1  - Production (Initial Inventory is zero)**

WA1 + WB1 + WOA1 + WOB1 ≥ 70

GA1 + GB1 + GOA1 + GOB1 ≥ 200$

FA1 + FB1 + FOA1 + FOB1 ≥ 140

**Period 2 – Production**

**Sum of Production in Period 2 and Inventory from Period 1** ≥ **Demand for Period 2**

WA2 + WB2 + WIA1 + WIB1 + WOA2 + WOB2 ≥ 125 + WA1

GA2 + GB2 + GIA1 + GIB1 + GOA2 + GOB2 ≥ 300 + GA1

FA2 + FB2 + FIA1 + FIB1 + FOA2 + FOB2 ≥ 175 + FA1

WIA1 + GIA1 + FIA1 ≤ 70 {Product inventory total max of 70 units}

WIB1 + GIB1 + FIB1 ≤ 50 {Product inventory total max of 50 units}

**Period 3 – Production**

**Sum of Production in Period 3 and Inventory from Period 2** ≥ **Demand for Period 3**

WA3 + WB3 + WIA2 + WIB2 + WOA3 + WOB3 ≥ 185 + WA2

GA3 + GB3 + GIA2 + GIB2 + GOA3 + GOB3 ≥ 295 + GA2

FA3 + FB3 + FIA2 + FIB2  + FOA3 + FO3 ≥ 205 + FA2

WIA2 + GIA2 + FIA2 ≤ 70 {Product inventory total max of 70 units}

WIB2 + GIB2 + FIB2 ≤ 50 {Product inventory total max of 50 units}

**Period 4 – Production**

**Sum of Production in Period 4 and Inventory from Period 3** ≥ **Demand for Period 4**

WA4 + WB4 + WIA3 + WIB3  + WOA4 + WOB4 ≥ 190 + WA3

GA4 + GB4 + GIA3 + GIB3 + GOA4 + GOB4 ≥ 245 + GA3

FA4 + FB4 + FIA3 + FIB3  + FOA4 + FOB4 ≥ 235 + FA3

WIA3 + GIA3 + FIA3 ≤ 70 {Product inventory total max of 70 units}

WIB3 + GIB3 + FIB3 ≤ 50 {Product inventory total max of 50 units}

**Period 5 – Production**

**Sum of Production in Period 5 and Inventory from Period 4** ≥ **Demand for Period 5**

WA5 + WB5 + WIA4 + WIB4  + WOA5 + WOB5 ≥ 200 + WA4

GA5 + GB5 + GIA4 + GIB4 + GOA5 + GOB5 ≥ 240 + GA4

FA5 + FB5 + FIA4 + FIB4 + FOA5 + FOB5 ≥ 230 + FA4

WIA4 + GIA4 + FIA4 ≤ 70 {Product inventory total max of 70 units}

WIB4 + GIB4 + FIB4 ≤ 50 {Product inventory total max of 50 units}

WIA5 + GIA5 + FIA5 WIB5 + GIB5 + FIB5 ≤ 0 {Should have no inventory at the end}

| **Product** | **Sell for ($)** |
| --- | --- |
| Widgets | 2490 |
| Gadgets | 1990 |
| Flugels | 2970 |

| **Product** | **Each $ invested creates one unit of additional demand for the next period** |
| --- | --- |
| Widgets | 160 |
| Gadgets | 120 |
| Flugels | 180 |
|  | Total Advertising Budget: $70,000 |

**Advertising Budget**

160(WA1 + WA2 +WA3 +WA4) + 120(GA1 +GA2 + GA3 + GA4) + 180(FA1 + FA2 + FA3 + FA4 ) ≤ 70,000

***Plant A:***

| **Product** | **Raw Material 1 (lbs)** | **Raw Material 2 (lbs)** | **Labor (hours)** |
| --- | --- | --- | --- |
| Widgets | 194 | 8.6 | 9.5 |
| Gadgets | 230 | 0 | 7.1 |
| Flugels | 178 | 11.6 | 11.1 |

Labor is limited to 2500 hours in each period

Overtime can be scheduled in any amount if necessary

| *Labor Costs* | **Period 1** | **Period 2** | **Period 3** | **Period 4** | **Period 5** |
| --- | --- | --- | --- | --- | --- |
| Regular ($/hour) | 11 | 11 | 11.55 | 11.55 | 11.55 |
| Overtime ($/hour) | 16.50 | 16.50 | 17.325 | 17.325 | 17.325 |

**Plant A Labor Hours:**

9.5\*WA1 + 7.1\*GA1 + 11.1\*FA1 ≤ 2500 {Labor limited to 2500 hours each period}

9.5\*WA2 + 7.1\*GA2 + 11.1\*FA2 ≤ 2500 {Labor limited to 2500 hours each period}

9.5\*WA3 + 7.1\*GA3 + 11.1\*FA3 ≤ 2500 {Labor limited to 2500 hours each period}

9.5\*WA4 + 7.1\*GA4 + 11.1\*FA4 ≤ 2500 {Labor limited to 2500 hours each period}

9.5\*WA5 + 7.1\*GA5 + 11.1\*FA5 ≤ 2500 {Labor limited to 2500 hours each period}

| **Product** | **Inventory Costs**  (Store from 1 Period to the Next) |
| --- | --- |
| Widgets | 7.50 |
| Gadgets | 5.50 |
| Flugels | 6.50 |

The product inventory area can store a combined maximum of 70 units

***Plant B:***

| **Product** | **Raw Material 1 (lbs)** | **Raw Material 2 (lbs)** | **Labor (hours)** |
| --- | --- | --- | --- |
| Widgets | 188 | 9.2 | 9.1 |
| Gadgets | 225 | 0 | 7.8 |
| Flugels | 170 | 10.8 | 10.6 |

Labor is limited to 3800 hours in each period

Overtime can be scheduled in any amount if necessary

**Plant B Labor Hours:**

9.1\*WA1 + 7.8\*GA1 + 10.6\*FA1 ≤ 3800 {Labor limited to 3800 hours each period}

9.1\*WA2 + 7.8\*GA2 + 10.6\*FA2 ≤ 3800 {Labor limited to 3800 hours each period}

9.1\*WA3 + 7.8\*GA3 + 10.6\*FA4 ≤ 3800 {Labor limited to 3800 hours each period}

9.1\*WA4 + 7.8\*GA4 + 10.6\*FA4 ≤ 3800 {Labor limited to 3800 hours each period}

9.1\*WA5 + 7.8\*GA5 + 10.6\*FA5 ≤ 3800 {Labor limited to 3800 hours each period}

| *Labor Costs* | **Period 1** | **Period 2** | **Period 3** | **Period 4** | **Period 5** |
| --- | --- | --- | --- | --- | --- |
| Regular ($/hour) | 11 | 11 | 12.10 | 12.10 | 12.10 |
| Overtime ($/hour) | 16.50 | 16.50 | 18.15 | 18.15 | 18.15 |

| **Product** | **Inventory Costs**  (Store from 1 Period to the Next) |
| --- | --- |
| Widgets | 7.80 |
| Gadgets | 5.70 |
| Flugels | 7.00 |

The product inventory area can store a combined maximum of 50 units

***Raw Material:***

| **Raw Material** | **Maximum Available (lbs)** | **Delivery Cost to Plant A ($/lb)** | **Delivery Cost to Plant B ($/lb)** |
| --- | --- | --- | --- |
| 1 | 140,000 | 1.25 | 1.45 |
| 2 | 5,000 | 2.65 | 2.90 |

Each plant only purchases raw material that can be used within a given period, since storage areas are limited.

**Raw Material 1:**

194\*WA1 + 188\* WB1 + 230\*GA1 + 225\* GB1 + 178\*FA1 + 170\* FB1 ≤ 140,000   {70 tons per period}

194\*WA2 + 188\* WB2 + 230\*GA2 + 225\* GB2 + 178\*FA2 + 170\* FB2 ≤ 140,000   {70 tons per period}

194\*WA3 + 188\* WB3 + 230\*GA3 + 225\* GB3 + 178\*FA3 + 170\* FB3 ≤ 140,000   {70 tons per period}

194\*WA4 + 188\* WB4 + 230\*GA4 + 225\* GB4 + 178\*FA4 + 170\* FB4 ≤ 140,000   {70 tons per period}

194\*WA5 + 188\* WB5 + 230\*GA5 + 225\* GB5 + 178\*FA5 + 170\* FB5 ≤ 140,000   {70 tons per period}

**Raw Material 2:**

8.6\*WA1 + 9.2\* WB1 + 11.6\*FA1 + 10.8\* FB1 ≤ 5,000                                  {2.5 tons per period}

8.6\*WA2 + 9.2\* WB2 + 11.6\*FA2 + 10.8\* FB2 ≤ 5,000                                  {2.5 tons per period}

8.6\*WA3 + 9.2\* WB3 + 11.6\*FA3 + 10.8\* FB3 ≤ 5,000                                  {2.5 tons per period}

8.6\*WA4 + 9.2\* WB4 + 11.6\*FA4+ 10.8\* FB4 ≤ 5,000                                  {2.5 tons per period}

8.6\*WA5 + 9.2\* WB5 + 11.6\*FA5+ 10.8\* FB5 ≤ 5,000                                  {2.5 tons per period}

***Transportation Costs***

| **Product** | **Plant A** | **Plant B** |
| --- | --- | --- |
| Widgets | $6.30 | $6.50 |
| Gadgets | $4.60 | $5.00 |
| Flugels | $5.50 | $5.70 |

There is an average transportation cost associated with the shipment of each unit of finished product from each plant to the Distribution Center.

Bibliography

Bhatia, Mahak.2020. “Linear Programming Approach- Application in Agriculture”*.* AccessedDecember 3,2021. <https://www.researchgate.net/publication/338395440_Linear_Programming_Approach-_Application_in_Agriculture>

Boren,Nykvist. n.d. “ Optimization of inventory usage”*.* Accessed November 29,2021. <http://www.it.uu.se/edu/course/homepage/projektTDB/ht18/project22/Project22_report.pdf>

Karthe.2017. “Introductory guide on Linear Programming for (aspiring) data scientists”. Analytics Vidhya. Accessed December 3,2021. <https://www.analyticsvidhya.com/blog/2017/02/lintroductory-guide-on-linear-programming-explained-in-simple-english/>

Kostreva, M., & Ogryczak, W. 1999. “Linear optimization with multiple equitable criteria. RAIRO “ Operations Research*,*Volume 33 , Issue 3 ,pp. 275 - 297. Accessed on December 2,2021.

Li. n.d “Large Scale Linear Programming-Solving Two-Stage Stochastic Programs: A Benders Decomposition Based Approach”. Accessed December 1,2021. <https://personal.utdallas.edu/~bxl180002/courses/OR705_Report_Li.pdf>

Maxime C. Cohen, Ngai-Hang Zachary Leung, Kiran Panchamgam, Georgia Perakis and Anthony Smith. 2017. “The Impact of Linear Optimization on Promotion Planning”. Accessed on December 5,2021.<https://doi.org/10.1287/opre.2016.1573>

Pap, Z. (2008, September). “Crop rotation constraints in agricultural production planning. In Intelligent Systems and Informatics, Crop rotation constraints in agricultural production planning. In Intelligent Systems and Informatics. SISY 2008”.6th International Symposium on (pp. 1-5) Accessed on Dec 3rd, 2021

Zukui Li, Ran Ding, and Christodoulos A. Floudas. 2011. “A Comparative Theoretical and Computational Study on Robust Counterpart Optimization: I. Robust Linear Optimization and Robust Mixed Integer Linear Optimization*”*. Industrial & Engineering Chemistry Research*.* Accessed November 29,2021.<https://pubs.acs.org/doi/abs/10.1021/ie200150p>